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PART A: NOTES

CHAPTER 1
UNITS AND DIMENSIONS

1.1 DIMENSIONS AND SYSTEM OF UNITS

A *dimension* is a physical specification of a system (length, time, etc). There are primary dimensions and secondary dimensions.

A *primary dimension* is one which is arbitrarily defined. For example, one dimension is length which has units of foot. The foot was defined as the physical length of a king's foot; a rather arbitrary definition.

A *secondary dimension* is one which is defined in terms of primary dimensions; e.g., volume (secondary) is defined in terms of a cubic length (primary).

Units give the magnitude of some dimension relative to an arbitrary standard. For example, when we say that a person is six feet tall, we mean that person is six times as long as an object whose length is defined to be one foot.

Why do we need Units?

Units are important for effective communication and standardization of measurements.

1.2 FUNDAMENTAL AND DERIVED UNITS

- **Fundamental** dimensions / units are those that can be measured independently and are sufficient to describe essential physical quantities.

- **Derived** dimensions / units are those that can be developed in terms of the fundamental dimensions / units.

    - **Force:** Newton's third law states that force on an object is mass times acceleration

$$F = m \times a$$

Therefore, the unit of force (Newton) is the unit of mass times M (kg) times the unit of acceleration LT^{-2} (m / s^2).

- **Viscosity:** The unit of viscosity can be determined from Newton's law for viscosity for a fluid between two plates separated by a distance L having a relative velocity V as in figure 1.1.

$$\tau = \mu V/L$$

In the above equation, τ is the shear stress on the plates, and has dimensions of force per unit area $ML^{-1}T^{-2}$, the velocity has dimensions of LT^{-1} and distance between the plates has dimensions of L. Therefore, the viscosity has dimensions of $\tau L/V$, which is $ML^{-1}T^{-1}$.

- **Specific heat** The change in thermal energy ΔE is related to the change in temperature of an object ΔT as follows

$$\Delta E = mC\Delta T$$

In the above equation, ΔE has dimensions of energy ML^2T^{-2}, mass has dimension M and temperature has dimension Θ^{-1}. Therefore, the specific heat has dimension $L^2T^{-2}\Theta^{-1}$, or units ($m^2/s^2/K$).

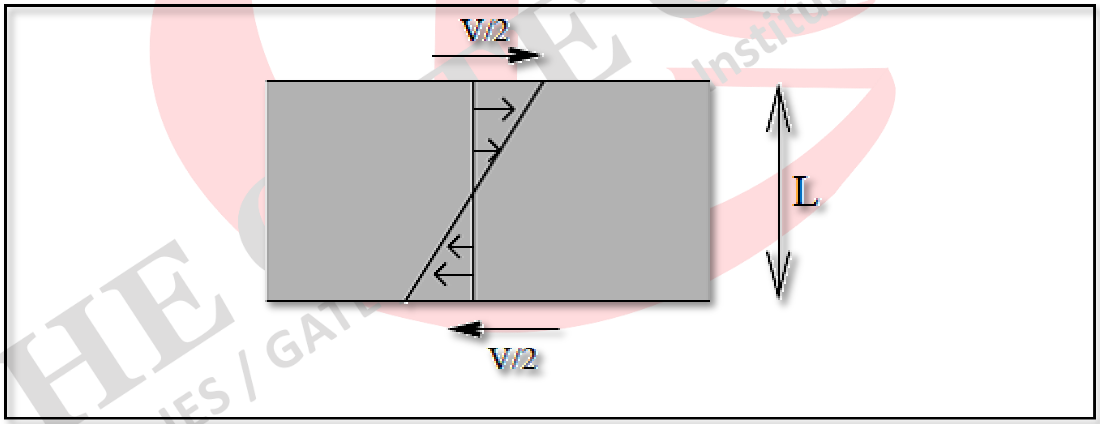

Figure 1.1: The fluid flow between two plates separated by a distance *L* moving with velocity (± *V*/2) in the tangential direction

The following table shows the list of basic, derived and alternative units in SI systems:

Physical Quantity	Name of Unit	Symbol for Unit	Definition of Unit
	Basic SI Units		
Length	Meter	m	
Mass	Kilogram	kg	
Time	Second	s	
Temperature	Kelvin	K	
Molar Amount	Mole	Mol	
	Derived SI Units		
Energy	Joule	J	$kg . m^2 . s^{-2} \rightarrow Pa . m^3$
Force	Newton	N	$kg . m . s^{-2} \rightarrow J . m^{-1}$
Power	Watt	W	$kg . m^2 . s^{-3} \rightarrow J . s^{-1}$
Density	Kilogram per cubic meter		$kg . m^{-3}$
Velocity	Meter per second		$m . s^{-1}$
Acceleration	Meter per second squared		$m . s^{-2}$
Pressure	Newton per square meter, pascal		$N . m^{-2}$, Pa
Heat Capacity	Joule per (kilogram . kelvin)		$J . kg^{-1} . K^{-1}$
	Alternative Units		
Time	minute, hour, day, year	min, h, d, y	
Temperature	degree Celsius	^0C	
Volume	litre (dm^3)	L	
Mass	tonne, ton (Mg), gram	t, g	

1.3 DIMENSIONAL CONSISTENCY

Dimensions and units must be handled consistently in any algebraic calculation. To be added, two quantities must have the same dimensions and units. (Adding a volume and a mass is guaranteed to be wrong.) The factors in a multiplication or division may have different units, and the combined quantity will have units of the product or ratio of the factors. Equations involving physical quantities must have the same dimensions on both sides, and the dimensions must be the correct ones for the quantity calculated. The units on both sides will usually also be the same, and must be at least equivalent and correct.

Verifying dimensional consistency is often called "checking the units," and is a powerful technique for uncovering errors in calculations. For purposes of checking consistency, dimensions or units may be considered algebraic quantities. Some examples of this procedure are:

• Density is defined as the ratio of mass to volume, and must have dimensions of mass / $(\text{length})^3$, with corresponding units.

• Checking dimensions for the famous formula $E = mc^2$

$$(\text{energy}) = (\text{mass})(\text{speed})^2$$
$$(\text{force})(\text{length}) = (\text{mass})(\text{length/time})^2$$
$$(\text{mass})(\text{acceleration})(\text{length}) = (\text{mass})(\text{length})^2/(\text{time})^2$$
$$(\text{mass})(\text{length/time})^2(\text{length}) = (\text{mass})(\text{length})^2/(\text{time})^2$$
$$(\text{mass})(\text{length})^2/(\text{time})^2 = (\text{mass})(\text{length})^2/(\text{time})^2$$

Hence, it is clear that the above equation is dimensionally consistence.

1.4 DIMENSIONAL EQUATIONS

A dimensional equation is one in which the units of measurement and their powers are used rather than their actual numeric values. For example, consider an object under constant acceleration: let *u* denote its initial velocity *v* denote its final velocity, *a* denotes the acceleration and *t* is the time between the initial and final points of time.

Then $v = u + a \times t$

The dimensional equation is

$$[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T]$$

Where, L represents a dimension of length,

T represents a dimension of time
M which does not appear here, would represent mass.

Only terms with the same dimensions may be added or subtracted.

1.5 CONVERSION FACTOR

For converting one set of units to another is simply to multiply any number and its associated units by ratios termed as *conversion factors* to arrive at the desired answer and its associated units.

Conversion factors are statements of equivalent values of different units in the same system or between systems of units used in the form of ratios.

E.g.

- Express a speed of 50 kilometers per hour as meters per second

$$50\ km/h = \frac{50km}{h}\frac{1000m}{km}\frac{1h}{60\min}\frac{1\min}{60s} = 14\ m/s$$

- Convert a concentration of 220 mg / dl to grams / liter

$$220\ mg/dl = \frac{220mg}{dl}\frac{1g}{1000mg}\frac{10dl}{l} = 2.20\ g/l$$

Example 1.1 In a multiple effect evaporator system, the second effect is maintained under vacuum of 475 torr, find the absolute pressure in kPa.

Solution: Absolute pressure = Atmospheric pressure – vacuum
= 760 – 475 = 285 torr

Absolute pressure $= 285\ torr \times \left(\frac{101.325\ kPa}{760\ torr}\right)$

= 38 kPa.

Example 1.2 Convert the 2 atm pressure into mmHg.

Solution:

Basis: 2 atm pressure

Conversion factor between atm and mmHg is : 1 atm = 760 mmHg

Thus

$$\text{Pressure} = 2(atm)\times\left(\frac{760}{1}\right)\left(\frac{mmHg}{atm}\right)$$

$$= \mathbf{1520\ mmHg}$$